**Section 21.150 Mathematics Standards for Mathematics Teachers in the Middle Grades**

In addition to the standards set forth in Subpart B of this Part, each mathematics teacher in the middle grades shall possess the knowledge and skills articulated in this Section.

a) Core Content Area Knowledge

1) Calculus

Effective middle grade mathematics teachers:

A) demonstrate knowledge of properties and notation of real numbers, properties of exponents and radicals, factoring techniques, solving polynomial equations and operations with rational expressions;

B) on the Cartesian Plane, graph polynomial, rational and radical functions and circles, and find horizontal and vertical asymptotes, and points of intersection of curves;

C) define function, domain, range, inverse functions, operate on functions, and use functional notation;

D) define one-sided, general and at infinity limits, and evaluate them by using the properties of limits;

E) define and apply the properties of continuous functions and determine discontinuities;

F) define first-order and higher-order derivatives and evaluate them using constant power, constant multiple, product, quotient and chain rules and by implicit differentiation;

G) apply the rules of derivatives to find tangent line, slope, rate of change, velocity and acceleration, marginal analysis, increasing and decreasing functions, curve sketching with maxima and minima and concavity, and solving optimization problems;

H) demonstrate knowledge of properties of exponential and logarithmic functions and their derivatives;

I) demonstrate knowledge of basic anti-derivatives, explore integration using the notion of "area under the curve" to determine definite integrals and understand the "Fundamental Theorem of Calculus" as a tool to evaluate definite integrals and relate integration and differentiation; and

J) apply the above knowledge and skills to applications from natural, physical and social sciences.

2) Statistics

Effective middle grade mathematics teachers:

A) construct, identify and interpret frequency distributions, histograms, cumulative frequency tables, ogives and box plots;

B) identify, calculate and interpret measures of central tendency and dispersion;

C) identify, calculate and apply the methods of counting;

D) identify, calculate and interpret probabilities and expected value;

E) define random variables and analyze and interpret the probability distributions they generate;

F) identify and describe the sampling distribution of sample means and sample proportions;

G) create and interpret confidence intervals for single population means and proportions;

H) identify, analyze and perform formal tests of hypotheses concerning single population means and single population proportions; and

I) identify, calculate and interpret the correlation coefficient and regression equations.

b) The Mathematics Curriculum

Effective middle grade mathematics teachers:

1) understand the Illinois Learning Standards for Mathematics (see 23 Ill. Adm. Code 1.Appendix D), their organization, progressions and the interconnections among the domains; and

2) know the developmental sequence of mathematics skills, along with age-level or grade-level benchmarks of development.

c) Foundational Knowledge

1) Standards for Mathematical Practice

Effective middle grade mathematics teachers enable students to acquire the skills necessary for strong mathematical practice in that they are able to:

A) make sense of problems and persevere in solving them;

B) reason abstractly and quantitatively;

C) construct viable arguments and critique the reasoning of others;

D) model with mathematics;

E) use appropriate tools strategically;

F) attend to precision;

G) look for and make use of structure; and

H) look for and express regularity in repeated reasoning.

2) Ratio and Proportional Relationships

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to ratio and proportional relationships and:

A) understand and apply fractions as numbers that can be modeled from a length perspective (number line), an area perspective (pattern blocks, geoboards, etc.), and a discrete perspective (set of dots or circles);

B) understand and apply the concept of unit fractions, benchmark fractions and the whole (referent unit) as defined in the Illinois Learning Standards for Mathematics;

C) extend the associated meanings of the properties of operations from whole numbers to fractions;

D) understand and use equivalent fractions, including those of whole numbers, to reveal new information and as a tool for comparison or to perform operational procedures;

E) understand and apply the connection between fractions and division, and how fractions, ratios and rates are connected via unit rates, and solve problems and formulate equations for proportional relationships;

F) describe the relationship between fractions and terminating, periodic and delayed-periodic decimals;

G) reason about how quantities vary together in a proportional relationship, using tables, double number lines and tape diagrams as supports;

H) distinguish proportional relationships from other relationships, such as additive relationships and inversely proportional relationships; and

I) understand the connection between a proportional relationship and a linear relationship.

3) The Number System

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to the number system and:

A) understand how the place value system relies on repeated groupings of any fixed natural number quantity (including ten) and can show how to use objects, drawings, layered place value cards and numerical expressions to help reveal place value structure, and extend place value system knowledge to negative, rational, irrational and real numbers;

B) efficiently use place value computation methods for addition, subtraction, multiplication and division with an understanding of composing and decomposing numbers using the commutative, associative and distributive properties, and, using multiple models, explain why rules for multiplying and dividing with negative numbers make sense;

C) derive various (multiple) algorithms and recognize these as summaries of reasoning, rather than rules, and distinguish between and understand the appropriate use of computation strategies and computation algorithms as defined in the Illinois Learning Standards for Mathematics, recognizing the importance of "mental math";

D) understand and explain methods of calculating products and quotients of fraction, by using area models, tape diagrams and double number lines, and by reading relationships of quantities from equations;

E) understand the concepts of greatest common factor, least common multiple, units, scale, origin, quantities, integer exponents, rational exponents, irrational numbers, complex numbers and radicals; and

F) understand the connections between fractions and decimals, particularly with regard to decimal computations.

4) Expressions and Equations

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to expressions and equations and:

A) understand operations and their associated inverses, and use properties of operations to rewrite polynomial expressions to reveal new information and to solve equations;

B) illustrate the meaning of 0 and why division by 0 leads to an undefined answer;

C) explain each step in solving an equation as following from the equality asserted at the previous step, while using the equal sign appropriately;

D) create and solve, using multiple representations, one-variable and two-variable equations and inequalities with letters representing an unknown quantity, defining constraints as necessary, and understand and illustrate what it means to be a solution of one‑variable and two-variable equations and inequalities;

E) use the structure of an expression to identify ways to rewrite it, and choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression;

F) strategically use algebraic tools, such as tape diagrams, number lines, double number lines, graphing calculators and computer algebra systems, to solve problems and connect the strategy for the solution to standard algebraic techniques;

G) validate or dismiss the chains of reasoning used to solve equations and systems of equations;

H) understand proportional relationships and arithmetic sequences as special cases of linear relationships;

I) derive and justify multiple forms for the equations of non-vertical lines; and

J) understand and apply properties of integer exponents and radicals to generate equivalent numerical expressions and solve problems.

5) Geometry

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to geometry and:

A) compose and decompose shapes, classify shapes into categories and justify the relationships within and between the categories, and summarize and illustrate the progression from visual to descriptive to analytic to abstract characterizations of shapes;

B) use multiple models to informally explain and prove geometric theorems about angles, angle relationships, parallel and perpendicular lines, circles, parallelograms and triangles, including the Pythagorean theorem and its converse;

C) describe the connections (relationships) between geometric properties and arithmetic and algebraic properties, including proportional relationships, and adapt a problem in one domain to be solved in the other domain;

D) use the coordinate plane to reason about spatial locations, graph shapes and solve problems;

E) derive area formulas, such as the formulas for areas of triangles and parallelograms, considering the different height and base cases, including oblique cases;

F) demonstrate an understanding of dilations, translations, rotations and reflections, and combinations of these using dynamic geometry software and constructions;

G) understand congruence in terms of translations, rotations and reflections; understand similarity in terms of translations, rotations, reflections and dilations; solve problems involving congruence and similarity in multiple ways; and explain the criteria for triangle congruence and apply the congruence properties to prove geometric theorems and properties; and

H) understand area and volume, and give rationales for area and volume formulas that can be obtained by compositions and decompositions of unit squares or unit cubes, and solve real-world problems involving area, volume and surface area of any two-dimensional or three-dimensional shape.

6) Statistics and Probability

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to statistics and probability and:

A) use data displays to ask and answer questions about data in real-life situations and demonstrate an understanding of measures used to summarize data, including but not limited to, shape, center, mean, median, interquartile range, mean absolute deviation, spread and standard deviation;

B) examine the distinction between categorical and numerical data, reason about data displays and recognize the connection to statistical variability and distributions;

C) develop an understanding of statistical variability and its sources, and the role of randomness in statistical inference;

D) explore and explain relationships between two variables by studying patterns in bivariate data and two-way frequency tables;

E) use technology, including calculators, spreadsheets and tables, to create scatter plots, linear models, dot plots, histograms and box plots, as well as calculate correlation coefficients of data; and

F) calculate theoretical and experimental probabilities of simple and compound events, and understand why their values may differ for a given event in a particular experimental situation.

7) Functions

Effective middle grade mathematics teachers are prepared to develop student proficiency and address common misconceptions related to functions and:

A) define and use appropriately the concepts of function, input, output, domain, range, rate of change, intercept, interval, end behavior, function notation, relative maximum and minimum, symmetry, zeros, graphical transformation, recursive formula, explicit formula, arithmetic and geometric sequence.

B) examine and reason about functional relationships represented using tables, graphs, equations and descriptions of functions in words, and translate between representations of graphs, tables, real-life situations or equations; and

C) examine the patterns of change in proportional, linear, inversely proportional, quadratic and exponential functions, and the types of real-world relationships these functions can model, and write expressions, equations and/or functions based on these patterns.

d) Using High-Leverage Instructional Practices

Effective middle grade mathematics teachers:

1) choose and use mathematical tasks that entail complex mathematical work, build basic skills and allow for multiple answers or methods;

2) teach and use the content-specific language of mathematics;

3) lead whole-class math discussions (e.g., math talks) that engage all learners;

4) respond productively to student "errors" by probing the underlying thinking and providing targeted feedback;

5) appraise, choose and modify tasks and texts for a specific learning goal;

6) use specific mathematically focused positive reinforcement;

7) use public recording (e.g., posters, whiteboard) to collect and probe mathematical thinking (e.g., demonstrating multiple answers and methods; exploring when an algorithm may be the best solution and when another approach may provide a more efficient solution);

8) diagnose common (and not so common) patterns of student thinking; and

9) assess students' mathematical proficiency and teach responsively.

e) Using Materials, Tools and Technology

Effective middle grade mathematics teachers:

1) apply mathematical content and pedagogical knowledge to select and use instructional tools, such as manipulatives and physical models, drawings, virtual environments, spreadsheets, presentation tools, websites and mathematics-specific technologies (e.g., graphing tools, interactive geometry software), recognizing both the insight to be gained and any limitations;

2) empower students to make sound decisions about the appropriate use of mathematical tools;

3) when making mathematical models, recognize that technology can enable one to visualize the results of varying assumptions, explore consequences, examine characteristics and compare predictions with data;

4) select mathematical examples that address the interests, backgrounds and learning needs of each student; and

5) evaluate curricular materials for appropriate level and depth of content, focus on and relevance to required learning goals and incorporation of the standards set forth in subsection (c)(1) of this Section.

f) Monitoring Student Learning through Assessment

Effective middle grade mathematics teachers:

1) engage in purposeful classroom assessment aligned to appropriate learning expectations for every student and monitor student progress in meeting developmental benchmarks in mathematics;

2) provide a variety of well-designed one-step, two-step, and complex multi-step assessment items and performance tasks that incorporate real-life situations, to allow students to demonstrate their learning;

3) ensure that assessments are responsive to, and respectful of, cultural and linguistic diversity and exceptionalities, and are not influenced by factors unrelated to the intended purposes of the assessment;

4) guide students in developing the skills and strategies for them to assess their work and set appropriate goals for their progress as mathematicians;

5) analyze student work to determine misunderstandings, misconceptions, predispositions and newly developing understandings, and use the results of this analysis to guide instruction and provide meaningful feedback; and

6) communicate the purposes, uses and results of assessments appropriately and accurately to students, parents and colleagues.

g) Meeting the Needs of Diverse Learners

Effective middle grade mathematics teachers:

1) understand the impact of cultural, linguistic, cognitive, academic, physical, social and emotional differences on mathematics development and progression of knowledge;

2) plan and implement mathematics instruction that capitalizes on strengths and is responsive to the needs of each student;

3) use a variety of approaches and classroom-based intervention strategies to respond to the needs of each student, particularly those who are struggling or advanced;

4) seek appropriate assistance and support for struggling and/or advanced learners;

5) collaborate and plan with other professionals to deliver a consistent, sequenced and supportive instructional program for each student;

6) differentiate strategies, materials, pace and levels of cognitive complexity to introduce concepts and skills to meet the learning needs of each student; and

7) make content accessible in appropriate ways to English language learners and students with exceptionalities.

h) Constructing a Supportive Mathematics Environment

Effective middle grade mathematics teachers:

1) create an environment that empowers every student to engage in the practices set forth in subsection (d) of this Part;

2) motivate and engage students by designing learning experiences that build self-direction, perseverance and ownership of mathematics;

3) guide students to work productively and collaboratively with each other to achieve mathematics learning goals by using a strategic combination of individual, group and whole class instruction to meet the learning needs of each student efficiently and effectively;

4) provide tools that are accessible and developmentally appropriate;

5) establish norms and routines for classroom discourse that allow for the respectful analysis of mistakes and the use of mathematical reasoning for mindful critique and argument; and

6) create opportunities and expectations that all students, including English language learners and students with exceptionalities, use appropriate written and oral mathematical language.

i) Professionalism, Communication and Collaboration

Effective middle grade mathematics teachers:

1) continually engage in intensive, ongoing professional growth opportunities that serve to increase mathematical knowledge for teaching, such as lesson study or continuing coursework;

2) use self-reflection to analyze instruction and make improvements and make use of strategies such as journal writing, video self-analysis and peer observation;

3) communicate and collaborate with other professionals, such as within a professional learning community, to plan teaching, discuss student needs, secure special services for students and manage school policies;

4) communicate and collaborate with families to support student needs and discuss student progress; and

5) maintain professional connections to improve mathematics instruction at local, State, regional and national levels.