**Section 373.APPENDIX B Modified Streeter-Phelps Equation**

The Modified Streeter-Phelps Equation mathematically defines the relationship between carbonaceous oxygen demand, nitrogenous oxygen demand, natural stream reaeration and the dissolved oxygen deficit as a function of time:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| D | = | KcLac | ( | e | -Kct | - | e | -K2t | ) |
| K2-Kc |  |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | + | KnLan | ( | e | -Kn(t-t0) | - | e | -K2(t-t0) | ) | + | Dae | -K2t |
| K2-Kn |  |  |  |

Definition and discussion of terms:

a) D = Dissolved oxygen deficit; units = mg/l; defined as the difference between the dissolved oxygen concentration at saturation and the actual instantaneous dissolved oxygen concentration at time t, i.e.,

 D = D.O.sat - D.O.actual. From this relation, the stream dissolved oxygen concentration can be computed for various times-of-travel (t's) downstream and plotted on a graph of t vs. D.O.actual.

b) Kc = carbonaceous decay constant; units = 1/day; this constant describes the rate at which carbonaceous BOD is utilized in a stream. Its value may be determined experimentally for a specific effluent and a specific stream. The actual value of Kc depends essentially upon the origin and strength of the wastewater, the type of treatment that wastewater has undergone, as well as various stream characteristics.

 The following guidelines may be used for selection of a Kc value for various applications:

1) Effluents containing up to and including 10 mg/l BOD5: 0.10

2) Effluents containing between 10 and 30 mg/l BOD5: 0.30

3) Virtually all effluents may be tested using an appropriate experimental procedure for a more precise determination of Kc.

c) Lac = ultimate carbonaceous demand; units = mg/l; this term may be calculated once the BOD5 and Kc are known by use of the following equation:

|  |  |  |
| --- | --- | --- |
| Lac | = | Ef |
| 1-e | -5Kc |

In this equation, Ef is the treatment works effluent BOD5.

d) K2 = stream reaeration constant; units = 1/day; this constant describes the rate at which atmospheric oxygen diffuses into the water of a flowing stream. Its value depends upon the hydraulic and geometric properties of the stream in question. Many investigators have developed equations to predict K2. The equation given below has been shown to yield results which best fit the field observations of many researchers over a wide variety of stream types:

|  |  |  |  |
| --- | --- | --- | --- |
| K2 | = | (110.5H + 0.5832V2) | (SV)0.375 |
| H2 |

In this equation, "H" is average depth of flow in feet, "V" is stream average velocity in feet per second, and "S" is the dimensionless parameter, stream slope, ft./ft. Velocity and average depth of flow may not be estimated but must be field measured at the 7-day 10-year low flow stream condition or computed from field measurements of stream geometry (cross sections and slopes using ordinary principles of open-channel hydraulics). Significant changes in stream geometry will change average velocity and average depth of flow. K2 must be computed for each stream segment as defined in Appendix C.

e) e = the Naperian logarithm base, dimensionless; e = 2.71828....

f) t = time; units = days.

g) Kn = nitrogenous decay constant; units = 1/day; this constant describes the rate at which nitrogenous BOD is utilized in a stream. Its value may be determined experimentally for a specific effluent and a specific stream. Previous experimental work has established a range of typical values for Kn of 0.25 to 0.37 per day with an average of 0.29 per day. It should be noted that the higher values of Kn yield generally more conservative results when applied to the Streeter-Phelps Equation.

h) Lan = ultimate nitrogenous demand; units = mg/l; this term may be calculated, once the initial ammonia nitrogen concentration is established, by use of the following formula:

|  |  |  |
| --- | --- | --- |
| Lan | = | 4.57 (Amm-N concentration in mg/l). |

1. t0 = nitrogenous lag time; units = days; when a waste contains both carbonaceous and nitrogenous oxygen demand, there is usually a time lag before the onset of nitrogenous oxygen demand. The time lag may typically vary from 0-10 days with its actual value dependent upon the complex chemical characteristics of the waste as well as various stream characteristics. The value of t0 may be experimentally determined where effluent or stream field measurements are practicable. In the case of well nitrified effluents, the value of t0 may generally be considered to be less than 1 day. Note that for t less than t0 the nitrogenous term,

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| KnLan | ( | e | -Kn (t-t0) | - | e | -K2 (t-t0) | ) |
| K2 - Kn |  |  |

does not enter into the calculation of D.

j) Da = initial dissolved oxygen deficit; units = mg/l. Determined by subtracting assumed effluent dissolved oxygen concentration of 6.0 mg/l from dissolved oxygen saturation value at the expected maximum stream temperature.

Temperature Adjustments

KC, K2, Kn and Lac are temperature dependent quantities. The values calculated in accordance with the above are 20 degree Celsius values. Since the saturation D.O. decreases with increasing temperature, it will be necessary to adjust the parameters KC, K2, Kn and Lac to reflect the expected maximum stream temperature condition. In the equations listed below, T is the expected maximum stream temperature in degrees Celsius.

a) Kc(T) = Kc x 1.047(T-20)

b) K2(T) = K2 x 1.024(T-20)

c) Kn(T) = Kn x 1.047(T-20)

d) Lac(T) = Lac (0.02T + 0.6)

Since the time of the year at which the 7-day 10-year low flow occurs typically varies from stream to stream, it is not possible to prescribe a uniform maximum temperature adjustment throughout the state. The maximum temperature should be ascertained from field measurements in the stream at the time of year at which 7-day 10-year low flow is expected to occur. IEPA ambient water quality monitoring network data are available for making such determination. This data may be obtained by contacting the Division of Water Pollution Control.

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